

# TOUCHMATH<sup>®</sup>



YOUR GUIDE TO  
**TEACHER TRAINING**



## A Legacy of Multisensory Math Success

For 50 years, TouchMath® has provided systematic and explicit multisensory math instruction grounded in the **Science of Math** to measurably accelerate student achievement.



## TOUCHMATH TEACHER TRAINING GUIDE

Standards Edition Units & Extra Practice Kits

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# The TouchMath Approach

TouchMath is an explicit, multisensory math program for students who struggle with foundational math and abstract concepts from preschool through transition. Addressing key foundational skills, numbers and operations, geometry, measurement and more, TouchMath provides a complete, differentiated math solution tailored to meet the unique and diverse needs of all students.

Aligned to state and extended standards, TouchMath is used in special education programs in self-contained, resource, and inclusion classrooms throughout the country and as an effective, supplemental math solution for Tier 2 and Tier 3 interventions. Informed by assessments and data, TouchMath provides a time saving, instructional solution designed to meet the needs of students in individualized, small group, and whole group settings.

The educators who have developed and expanded the TouchMath Program throughout the last 50 years have worked constantly to meet this goal. Every TouchMath unit, module, and activity sheet reflects their dedication. The success TouchMath has experienced is largely due to its multisensory approach and adherence to research-proven practices. It is unique in that it combines the use of the four major senses of seeing, saying, hearing, and touching with the research-proven



**Concrete-Representational-Abstract (CRA) learning continuum** by using the abstract numeral as a manipulative. The CRA continuum reflects the sequence of instructional and research-based best practices for how most students learn. This is important because it helps students represent numbers and develop number sense quickly – preparing them for future success in math by ensuring they develop a tangible understanding of the math concepts they need to learn and have multiple strategies for solving math problems.

In addition, TouchMath incorporates other research-proven strategies in the lessons and activities, including explicit, systematic instruction as it is what they know from the research, which is also known as I Do, We Do, You Do. Educators will also find multiple ways for students to access, engage, and express their learning, also known as Universal Design for Learning. Constant time delay and errorless learning are other strategies available for use with those students who need it.

We have made sure that TouchMath activities are meticulously scaffolded and provide all learners with ample opportunities for success. As each skill is mastered, the next simple step is introduced. The TouchMath activity sheets are clean and uncluttered with age-appropriate artwork. The variety, quantity, and quality of the activities provided make reinforcement immediately available on an as-needed basis. Teacher materials are straightforward and easy to use. Skills on each page are clearly defined, connected to the standards, and presented using specific methodology. Instructional strategies include real-world connections, books to read, digital tools and group and individual work and games. The materials are effective in large groups, small groups, or for individual learners. Students master each concept as they see it, say it, hear it, touch it, and learn it.



## Key Solution Features

### Standards Aligned

Meaningful access to state and extended standards.

### Differentiated

Accessible and adapted lessons and supporting materials.

### Progress Monitoring

Inform instruction and intervention with centralized progress monitoring.

### Compliance and Efficacy

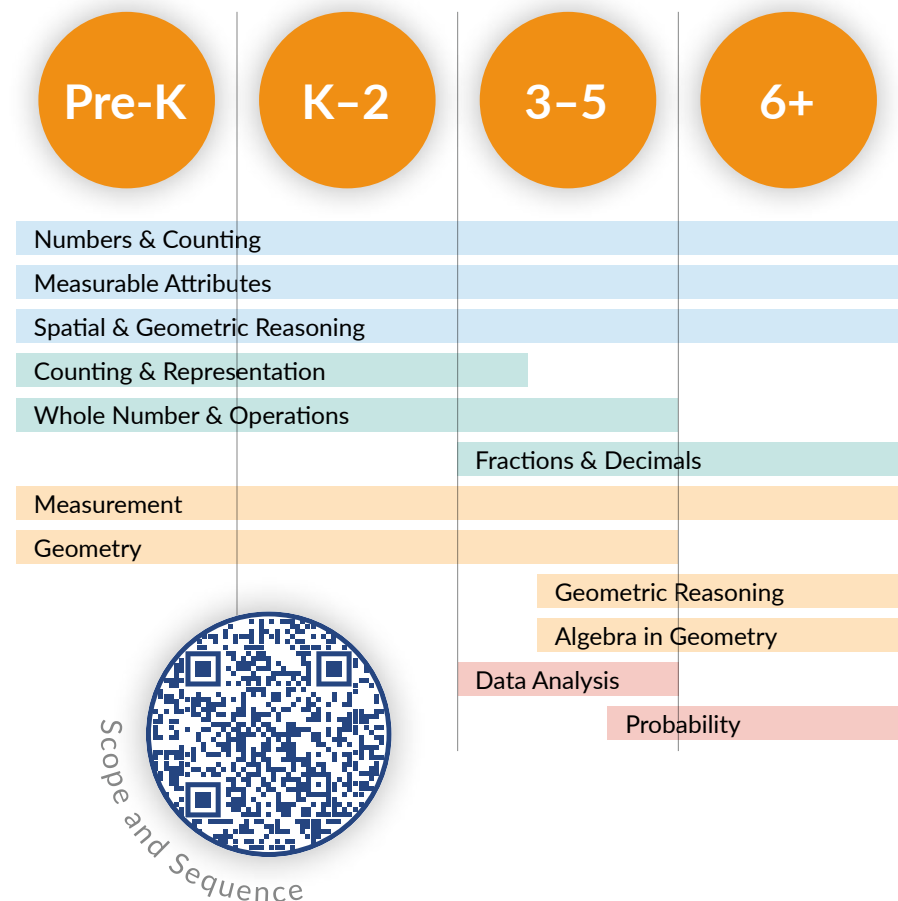
Maintain IEP compliance with streamlined processes.

### Enhanced Collaboration

Build positive relationships among educational teams and bridge home-to-school connections.

### Comprehensive Scope and Sequence

A detailed scope and sequence outlines the progression of math concepts and skills. Visit [touchmath.com/scopesequence](https://touchmath.com/scopesequence) or scan the QR code below to view the complete document.



TouchMath is committed to our mission of helping students conquer math. You can use the TouchMath program to give young children a head start, supplement a grade-level curriculum (Pre-K – Algebra), provide intervention, and achieve the best possible outcomes for any student.

# TOUCHMATH® Teacher Training Guide

## PRINT OR DIGITAL SOLUTION

### MyTOUCHMATH®



All Standards Edition (SE) curriculum units and Extra Practice (XP) kits are available in print or digital (print on demand) editions. No matter the edition, the exact same TouchMath content includes student activity sheets and teacher guide instruction.

MyTouchMath.com is now the ONE portal for students, teachers and Administrators to access TouchMath NOW, TouchMath HUB, and TouchMath PRO lessons. The digital environment lets students interact directly with on-screen content in both English and Spanish.

## PRINT SOLUTION

Designed specifically to help students build foundational math skills, TouchMath combines explicit and systematic instruction with multisensory strategies and specialized manipulatives to drive student outcomes. The print-based solution provides a standards-aligned multisensory program, informed by data and progress monitoring, and differentiated instruction specifically to help students who struggle with mathematics achieve positive outcomes.



### Print Solution Includes:

- ✓ Teacher Guides
- ✓ Student Activity Sheets
- ✓ Hands-On Manipulatives
- ✓ Progress Monitoring

[touchmath.com/print-solution](https://touchmath.com/print-solution)

## DIGITAL SOLUTION

Access the explicit, multisensory math program that is proven to remediate and accelerate student progress from one convenient, web-based platform. The TouchMath Digital Solution empowers educational teams with engaging digital tools designed to reduce workload, enhance instructional efficiencies, and significantly drive student outcomes.



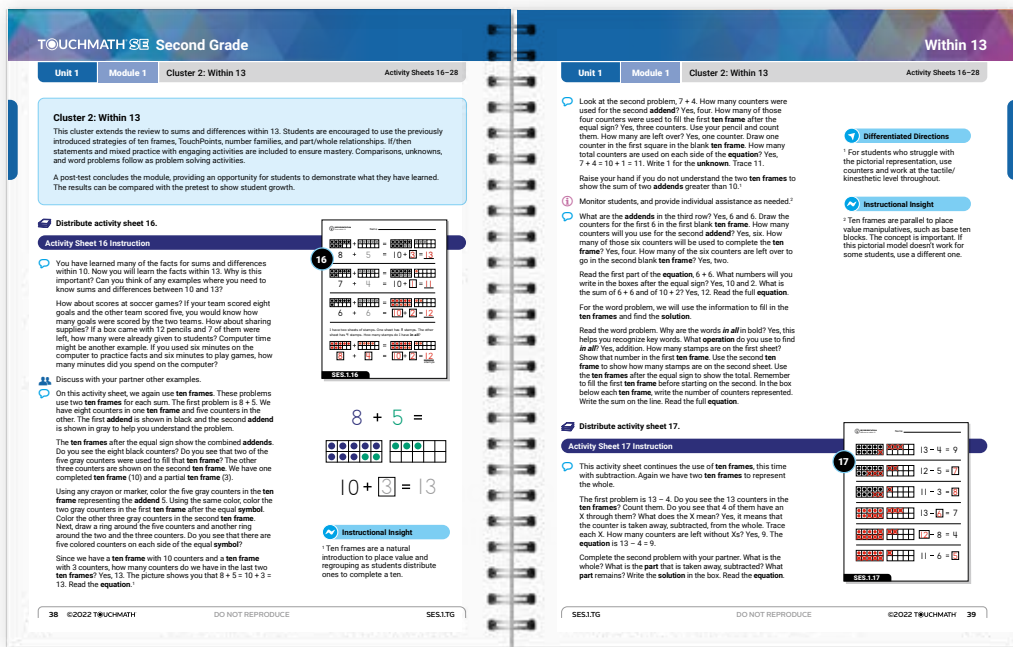
### Digital Solution Includes:

- ✓ Teacher and Student Licenses
- ✓ Web-based Curriculum and Resources
- ✓ Hands-On Manipulatives
- ✓ Digital Assessments and Progress Monitoring

[touchmath.com/digital-solution](https://touchmath.com/digital-solution)

When used independently, TouchMath NOW does not collect or store student personal information.

\*Student data privacy agreement (DPA) may be required.



## Teacher Guides

Each Teacher's Guide provides a consistent and predictable instructional sequence designed to remediate and accelerate student progress. Beginning with a pre-test, each unit of instruction is segmented into modules and clusters, providing explicit teacher-led instructions aligned to student activity sheets that scaffold based on the needs of each student.

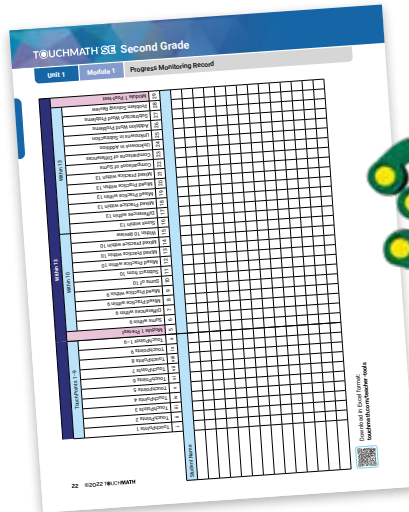
## Hands-On Manipulatives

Provide concrete, multisensory learning experiences with the classroom manipulative set included with each grade



## Explicit, Multisensory Instruction

Grounded within the CRA continuum, TouchNumerals and TouchPoints bridge physical objects to abstract representations in an explicit, yet meaningful way.



## Progress Monitoring

Pre- and post-assessments, ongoing data collection, and permanent work samples give educational teams the necessary data to inform instruction and drive student outcomes.



## Student Activity Sheets

Over 4,000 student activity sheets designed to meticulously work alongside our hands-on manipulatives support math development and skill generalization.



**Print Solution Add-Ons:** TouchMath's Award-Winning Student Digital Intervention. Classroom Manipulative Add-Ons.



### Classroom Manipulative Kit Included

See pages 7-8 for details

### Web-Based Curriculum

Everything you love about the Print Solution, and MORE! A Digital Solution subscription includes...

- Web-based Teacher Guides
- Printable Student Activity Sheets
- Classroom Manipulative Set
- Student Digital Intervention
- Digital Games and Practice
- Digital Student Portfolios
- Robust Reporting Capabilities

View Digital Solution system requirements at [touchmath.com/digital-solution](http://touchmath.com/digital-solution)



Available in Spanish

### Student Digital Intervention

TouchMath's award-winning digital intervention solution adapts to student responses using a proprietary algorithm that automates suggested math progressions on the visually distinct Skills Rainbow.

### Print and Digital Activities

Meet the needs of students on and off the computer with custom learning plans, interactive digital skills (number sense to algebra), and printable activity sheets – all available in English and Spanish.

### Digital Games and Practice

Increase engagement in-and-out of the classroom with digital skills practice and math games – all accessible on classroom interactive surfaces and personal devices.

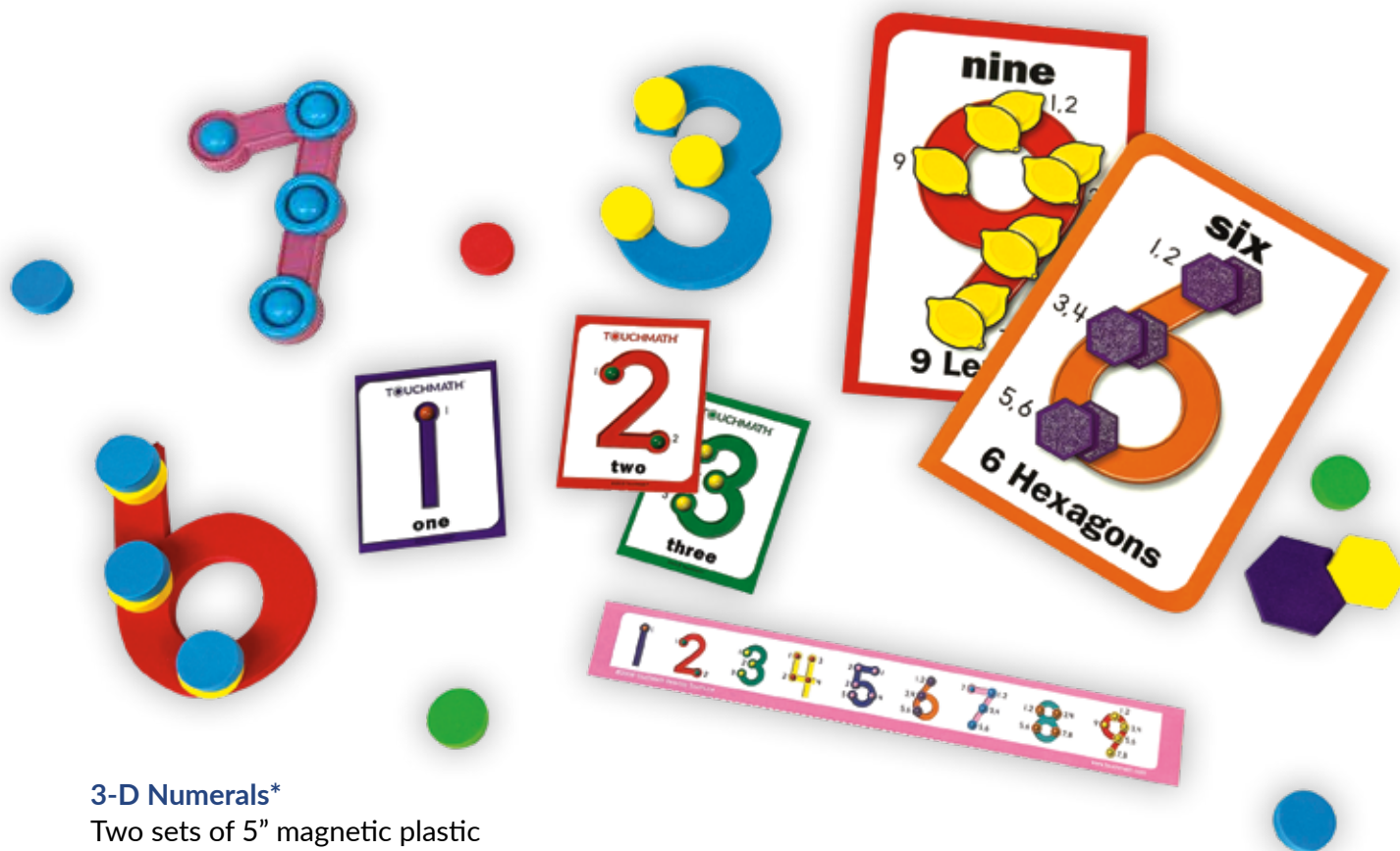


**Digital Solution Add-Ons:** Additional Student Licenses, Printed Teacher Guides, and Classroom Manipulative Add-Ons



## Classroom Manipulative Kit

Easily make abstract math concepts more accessible with multisensory math manipulatives. Included with both Print and Digital Solutions, TouchMath manipulatives are scientifically designed to correlate with student activity sheets and intervention components, providing a concrete, hands-on learning experience that is proven to accelerate learning.



### 3-D Numerals\*

Two sets of 5" magnetic plastic numerals and operation signs.

### TouchNumerals\*

Three sets of 5" foam numerals, detachable TouchPoints, and operation signs.

### Texture Cards

36 oversized cards with textured TouchPoints in four fun themes.

### TouchShapes\*

324-piece set including six durable plastic shapes in an assortment of colors and sizes.

### TouchPoint Counters\*

500 multi-colored, multi-use plastic counters.

### Desktop TouchLines

Set of 24 double-sided horizontal/vertical number lines with counting patterns.

### Student Number Cards

Set of 10 double-sided cards with/without counting patterns.

The Classroom Manipulative Set is included with all Print and Digital Solutions. The Pre-K Classroom Manipulative Set (not pictured) includes age-appropriate, TouchPoint Counters with Pictures. Manipulatives are not intended for use with children under 3 years of age. \*Includes downloadable content.



## Add-On Manipulatives

Enrich student learning and classroom experiences with additional hands-on manipulatives and instructional supports. Explore TouchMath's collection of student, teacher, and classroom resources that provide instructional support and hands-on learning opportunities.



### TouchPoint Counters with Pictures

Two sets (space and animals) of foam TouchPoints with colorful pictures.

### Domino Cards

Five sets of playing cards with textured pictures and TouchPoints.

### TouchCards

Double-sided fact mastery cards with/without textured TouchPoints.

- Addition • Subtraction
- Multiplication

### Computation Step Posters\*

Six 11x17 classroom posters showcase the TouchMath computational steps.

- Addition • Subtraction
- Multiplication • Division

### Skip Counting Posters & Songs\*

Eight 11x17 classroom posters and catchy songs for skip counting patterns 2–9. • Primary • Upper

### TouchPoint Posters\*

Nine double-sided wall or tabletop numeral posters with counting patterns.

- TouchPoints • With Pictures

### Counting Quantity Posters\*

Twelve colorful classroom posters for counting sequences 1–10, 20, and 30.

- Birds • Space

### Grade Level Workbooks

5-pack single-use student workbooks with teacher strategies. See website for grade and skill options.



## Opening Your Standards-Edition (SE) Box

Each SE print unit box contains double-sided reproducible activity masters, labeled tabs to keep the activity masters organized by module, and a spiral-bound Teacher's Guide. Unit boxes are designed for effortless organization, ease of use, and clutter-free storage when placed on a bookshelf or when stored in a drawer or filing cabinet.



The spiral-bound teacher's guide includes a comprehensive overview of the TouchMath program with page-by-page instruction for unit activity masters including answer keys, progress monitoring, and QR codes to quickly access training videos, state alignments, Scope & Sequence, and more. See page 5 to learn more.

TouchMath NOW customers will receive digital access to all of the same boxed activity sheet masters and Teacher's Guide support, accessible from any internet-connected computer or tablet device. See page 14 to learn more.

[Click box for more information](#)

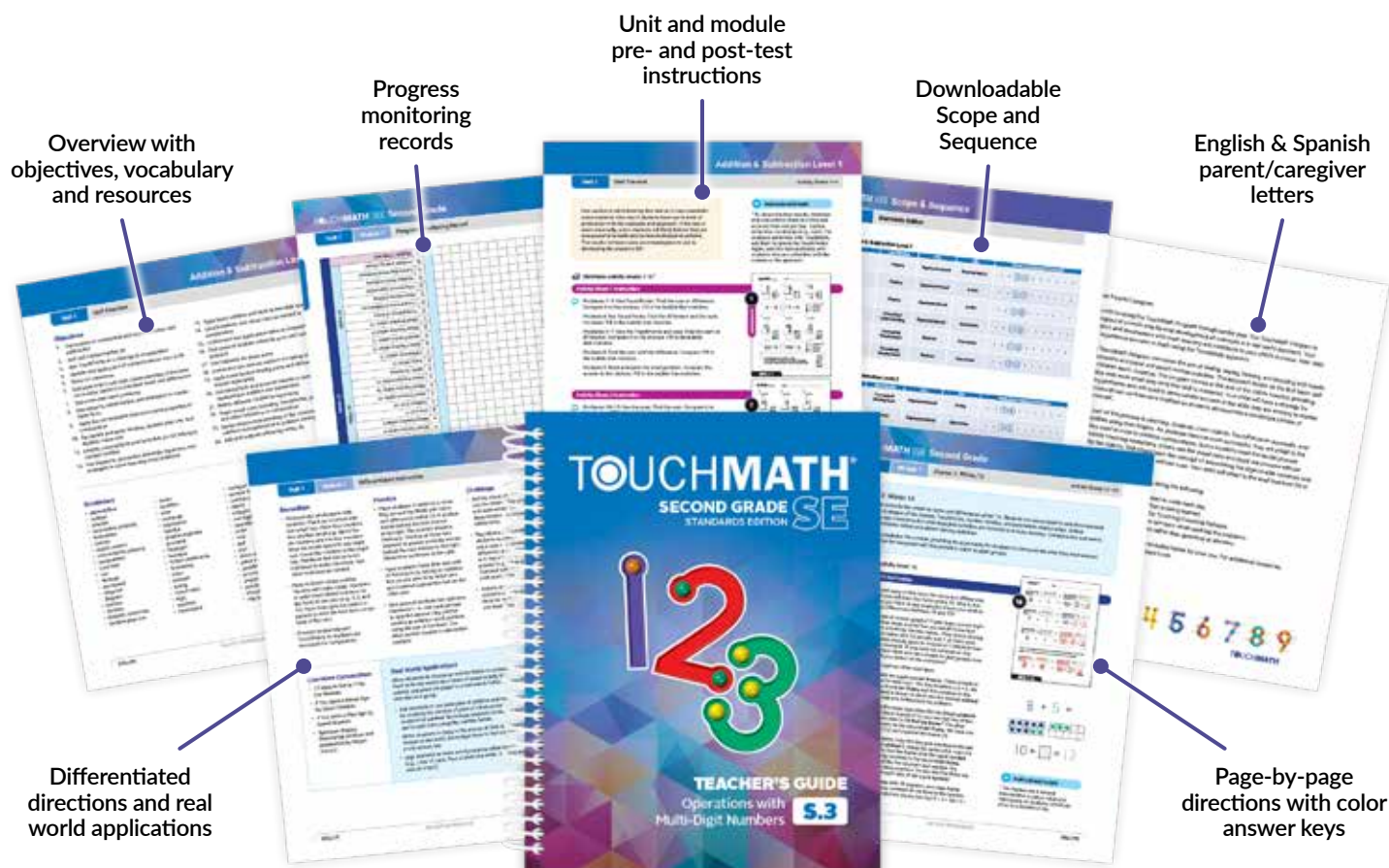


23 units make up the complete SE Curriculum: Pre-K (orange) 1 unit, Kindergarten (green) 4 units, First Grade (red) 4 units, Second Grade (blue) 4 units, and Upper Grades (purple) 10 units.

## A Look at the Teacher's Guide

Each SE print unit includes a spiral-bound teacher's guide (provided digitally for TouchMath NOW customers). Each teacher's guide summarizes the content for that unit and is broken into modules with page-by-page instruction for every student activity sheet. Also included are activity sheet thumbnails, a comprehensive color answer key, parent/caregiver communication letters in English/Spanish, and progress monitoring records for each module and assessment. These essential guides are organized for quick reference and easy use in a busy classroom. They contain the following:

- Unit and module overviews that provide the scope of content
- Find alignments to your state standards at [touchmath.com/state-standards-alignments](https://touchmath.com/state-standards-alignments)
- Comprehensive list of objectives and vocabulary covered in the unit
- Objectives that define a focus for the skills presented
- Prerequisites that identify experiences needed prior to the lessons
- Vocabulary that emphasizes teacher language to be used in concept building
- Materials that will be necessary for the lessons
- Page-by-page instructional strategies matched with student activity sheets
- Pre- and post-test assessment tools
- Parent/caregiver communication letters
- Progress monitoring records to track student progress and to plan differentiated instruction





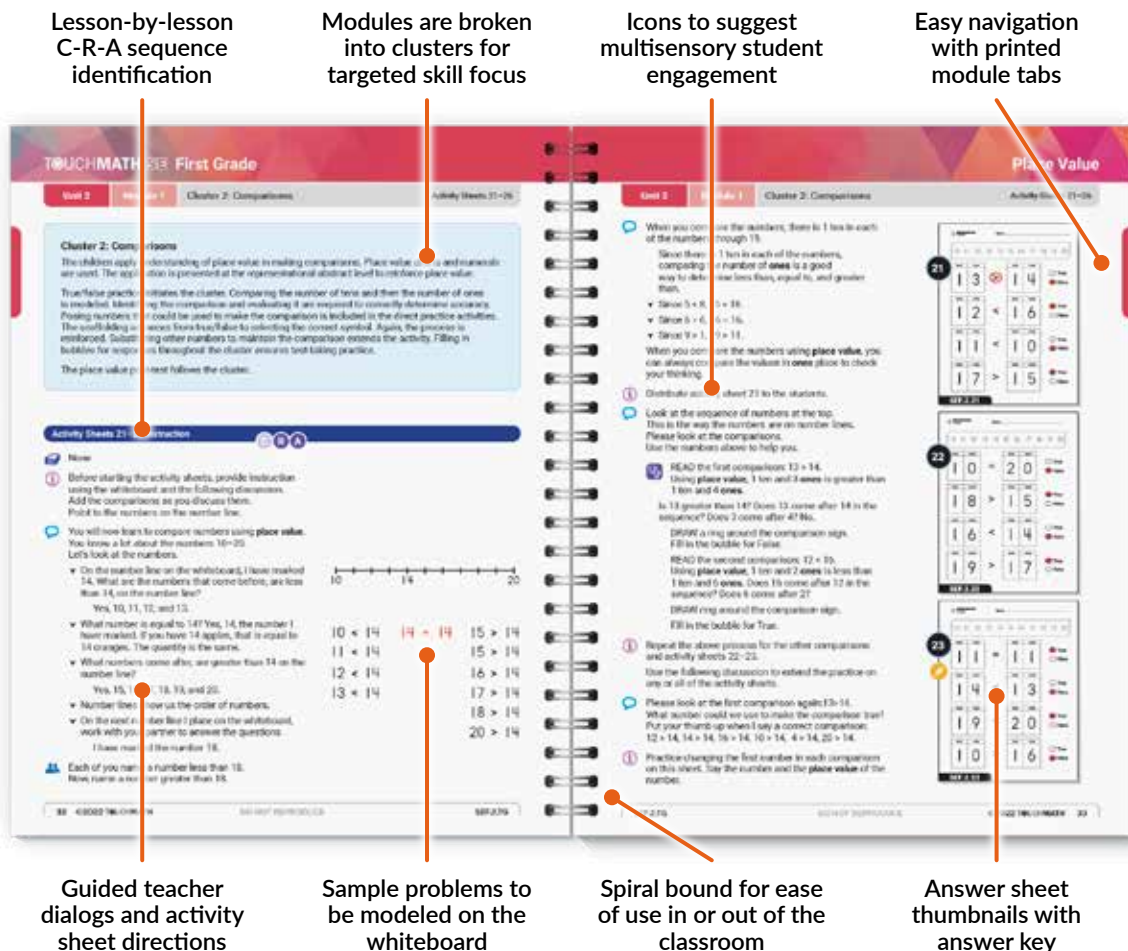
## Guided Instruction Visual Features

Each print teacher's guide includes page-by-page instructional strategies to be used with the student activity sheets. The two column presentation helps educators teach with ease by presenting easy to follow directions paired with activities and color answer keys.

The left column contains detailed page-by-page directions with teacher script, materials lists, bold vocabulary words, and icons for multisensory engagement, including:



The right column contains answer key thumbnails, activities, and whiteboard samples to be modeled and/or discussed. The Second Grade and Upper Grades programs also include basic background (information related to the skill), instructional insights (purpose of the strategy), and differentiated directions (suggestions for meeting various needs of individual learners).

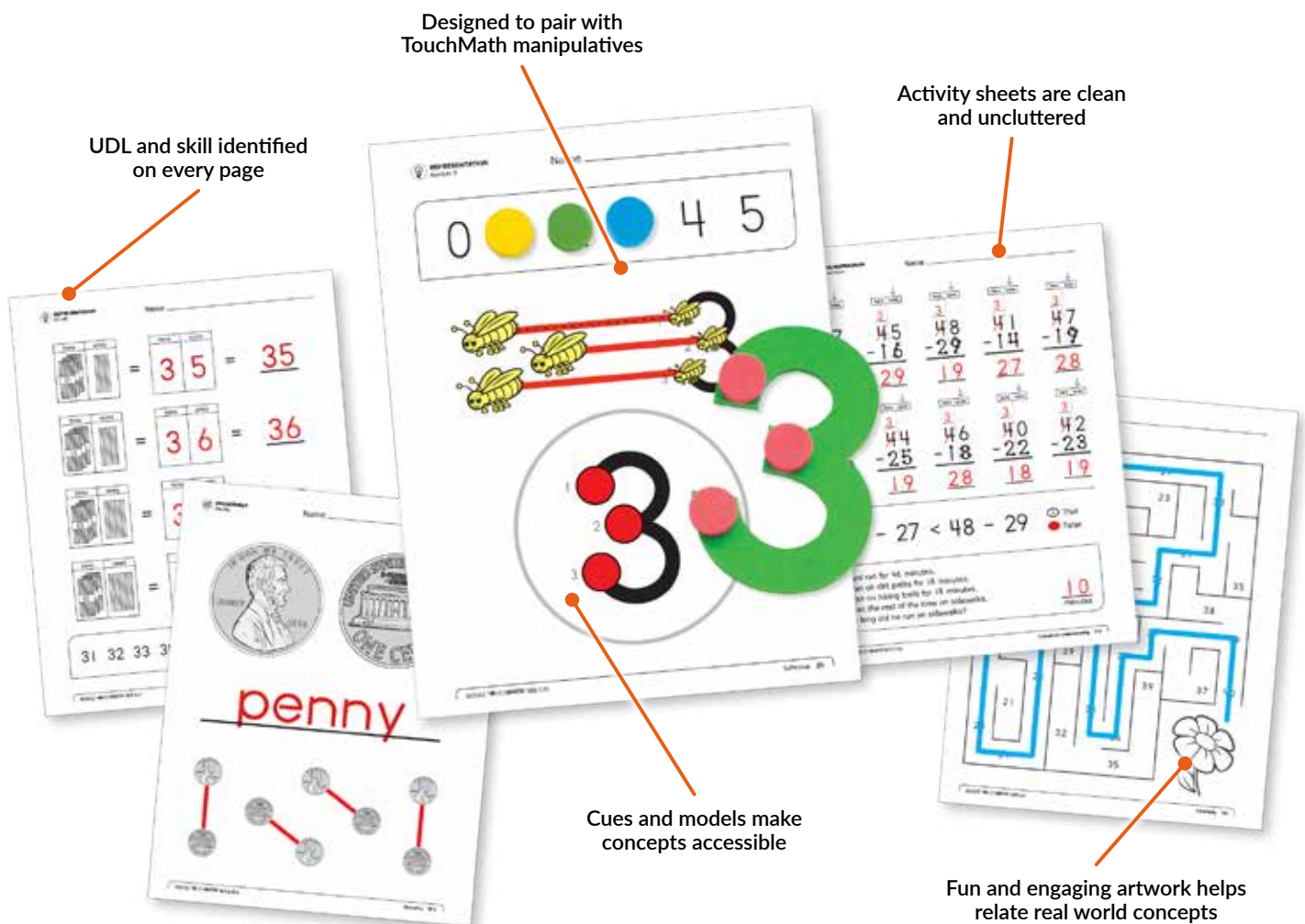


## SE Activity Sheets

Each SE unit contains reproducible (print) or downloadable (digital) student activity sheets, which are organized by skill into modules. These pages are aligned with current, rigorous state standards – and follow current state ESSA plans that mandate a culture of high expectations for all students. All skills are built upon the Concrete-Representational-Abstract (CRA) approach and Universal Design for Learning (UDL) framework, allowing all students to access a range of tools for building the foundational math skills they must master to succeed in critical math domains.

The activity sheets are designed to incorporate TouchMath manipulatives and common classroom materials where appropriate. They feature opportunities to discuss and build mathematical language naturally. The clean, uncluttered pages also allow students the necessary space to develop tactile skills as they master new concepts. As children see, say, hear, and touch while they practice math, they learn and remember.

Color thumbnail answer keys to all activity sheets can be found in the teacher's guide or TouchMath NOW platform alongside the instructional strategies.





# Math Screening Tool from TouchMath

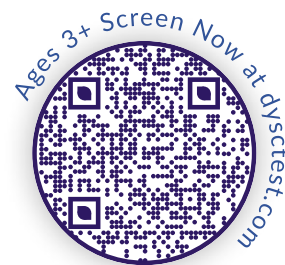
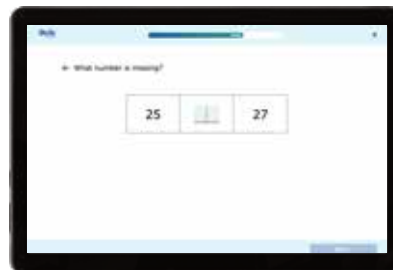
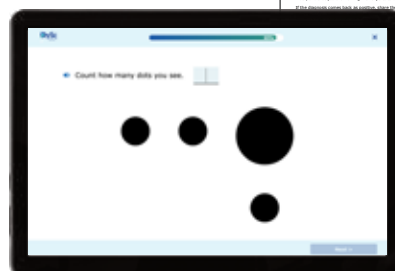
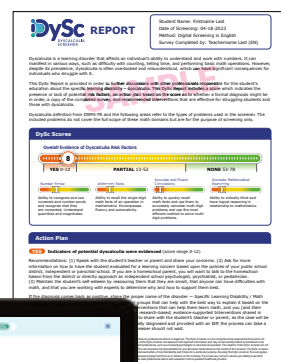
Dyscalculia is a neuro-developmental disorder that is estimated to affect 7% of the population. Similar rates to dyslexia, which affects reading ability, dyscalculia negatively impacts a child or adults' ability to attain number sense, memorize math facts, do accurate and fluent calculations, and can affect mathematical reasoning skills.



DySc (Dyscalculia Screener) is a screening tool developed and subsidized by TouchMath. The screener includes key questions to help identify weaknesses in the four key components of dyscalculia and math difficulties, and then provides a report (aligned to The Diagnostic and Statistical Manual of Mental Disorders, Fifth Edition (DSM-5) indicating the presence or lack of potential risk factors with an action plan for immediate intervention.

## To Screen Your Students:

1. Take the free screener at [dysctest.com](https://dysctest.com). The screening should take the student about 10 minutes to complete and can be taken digitally using a computer, tablet, or mobile device. You can also download a print version. The screening is available to students in both English and Spanish.
2. Once the screening is complete, please fill out the educator survey. Survey responses will be included in the report, providing valuable information to help a qualified professional determine whether a formal diagnosis might be in order.
3. The DySc report will be emailed to the teacher. Please allow 12 hours for delivery. The report will include a score which indicates the presence or lack of potential risk factors and an action plan based on the score. Early intervention is essential for struggling students – get started by using the included recommended interventions.
4. For a more robust intervention solution, contact your TouchMath sales consultant for information about the TouchMath Dyscalculia Intervention Extension, including 60 math games, 180 math activities, and intervention plans designed to help struggling students in the four key components of dyscalculia – all strategically aligned to the TouchMath Standards Edition curriculum.



## Dyscalculia Extension

Provide foundational support in the four key areas of dyscalculia. Includes two spiral bound teacher guides with page-by-page instructional support for 180 printable activity sheets, 60 math extension tasks/games, graphic organizers and visual supports, intervention plans, and progress monitoring.



[touchmath.com/curriculum-enhancements](https://touchmath.com/curriculum-enhancements)

# Welcome to the World of TouchMath Training

## Explicit Multisensory Math Instruction since 1975



*Since 1975, TouchMath has supported all students who struggle with math, those who struggle temporarily and those who struggle permanently.*

You are about to participate in an interactive Teacher Training that will guide you step-by-step through TouchMath computation. The program's benefits will become clear as you progress through the unique multisensory strategies.

TouchMath bridges the gap between concrete experiences and abstract concepts, reinforces number values, eliminates guessing, dramatically reduces student errors, and aids in memorization. But perhaps more important than all of those facts — is that by making math fun and accessible, TouchMath gives students confidence and inspires them to reach their academic potential.

This Manual is designed to be used in conjunction with the customized Professional Development Seminars, TouchMath Teacher Training videos, or as an independent reference resource. It includes step-by-step instructions that cover computational strategies ranging from the basic Touching/Counting Patterns and TouchPoint placement, to more common skills such as addition and subtraction, to fractions and pre-algebra.

Thank you for your interest in TouchMath, and for all you do to ensure students of all ability levels and learning styles find success in mathematics.

# Make It Easy On Yourself

Having the TouchMath Program in the classroom is one part of a successful overall strategy to implement quality math instruction. To ensure user fidelity and augment the effectiveness of the program, suggestions are included for classroom use.

## TouchMath Classroom Setup

- Display all TouchMath classroom aids during math instruction, and use them with the students.
- Provide centers featuring TouchNumerals, counters, and other manipulatives to reinforce learning.
- Place the progress monitoring records in an easily accessible place.
- Store the unit and/or kit boxes on a shelf for later use.
- Establish small group areas to differentiate instruction and provide individualized guidance.

## TouchMath Program Implementation

- Incorporate the objectives listed in the teacher's guides and/or implementation strategies into your lesson plans and/or IEPs.
- Include in your lessons the visual, auditory, and tactile/kinesthetic components mentioned in the instructional strategies.
- Model using TouchMath materials, manipulatives, and counters during whole group instruction.
- Use the vocabulary words included for each objective in your instruction with the children.
- Assess students at the end of each module to monitor progress.
- Send home the completed activity sheets to keep parents/guardians informed of the program material.
- Use opportunities outside of math instructional time (art, snack time, transitions) to reinforce TouchMath strategies and provide meaningful repetitions.
- Incorporate the skills in the TouchMath Program throughout the day.

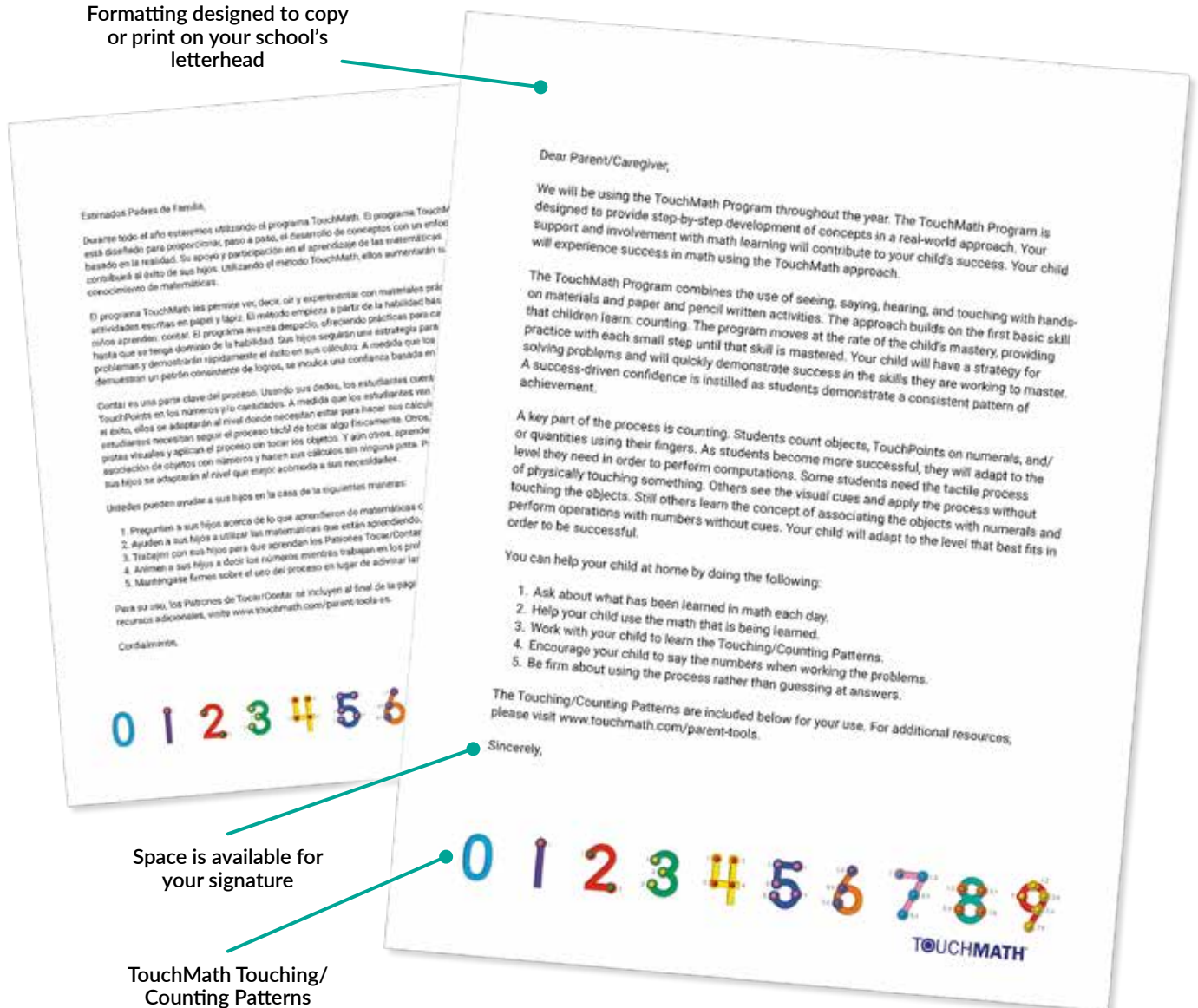
## TouchMath Program Implementation Tips

- **TouchPoints:** Provide explicit instruction to master the Touching/Counting Patterns.
- **Instructional Strategies:** Modify the teacher input/modeling, checking for understanding, and directions for guided practice, implementing those that work for you and your learners.
- **Activity Sheets:** Use only the activity sheets that are needed to advance the learning of individual students.
- **Practice:** Use enough activity sheets to provide meaningful repetition of the skill for the developmental level of the student.
- **Extra support:** Schedule a parent volunteer or paraprofessional to work with small groups or individuals who need more experience with the skill.
- **Classroom setup:** Locate details and examples in this implementation guide.
- **Amount of time:** In general, structure activities within the two to two-and-a-half times the learner's developmental age (e.g., If the student is 6 years old developmentally, activities will be most effective if they are 12 to 15 minutes long.). Independent time provides opportunities for additional experiences. Using one to four pages per day will provide ample opportunities for learning throughout the year.

## Connect School to Home

All Standards Edition units and Extra Practice kits include parent/caregiver communication letters in both English and Spanish. These letters can be found in the SE teacher's guides, in the XP kit introduction, or downloaded from the resource library section in TouchMath NOW. The introductory letter informs parents that TouchMath will be used with students during the school year. The letters are designed to be printed on school letterhead and signed by the teacher. Letters are also available on the [Teacher Tools](#) page of the website. In addition to an overview of the TouchMath approach, letters suggest ways parents can help at home, and include the Touching/Counting Patterns to help parents better support their child.

Formatting designed to copy  
or print on your school's  
letterhead





# Transitioning from Concrete to Symbolic Learning

Eminent learning theorists Jerome Bruner and Jean Piaget concluded that there are three major learning stages in early childhood development: concrete, pictorial and symbolic. Children must be in a state of readiness to absorb material on each of these levels.

TouchMath is designed to help students transition from concrete to symbolic learning. Young students first learn to count using familiar objects like apples, pencils, balloons, etc. Then we place pictures of these objects on the numerals. The objects are later replaced by TouchPoints. Eventually, most students progress to a symbolic understanding and leave the TouchPoints behind.

## CRA: Concrete-Representational-Abstract

The TouchMath approach increases student engagement and understanding by utilizing auditory, visual, and kinesthetic strategies. As students see, say, hear and touch the TouchPoints on numerals, they easily make the connection between the numeral and the quantity it represents. Decades of research have shown that TouchMath accelerates student progress, improves accuracy, and increases fluency. Additionally, research shows that the TouchMath approach is self-fading and increases student confidence.

Classification	Cardinality	Place Value	Real-World Problem Solving	Mixed Operations
Counting	Geometry	Backward Counting	Beginning Multiplication	Algebra
Shapes	Sorting	Time & Money	Division	Data Analysis
	Composing & Decomposing Numbers	Mixed Addition & Subtraction	Fractions & Decimals	Formulas
			Estimation	



## METHODOLOGY

# Mathematical Terminology

## Reach All Learners

Children learn most effectively when all of their senses are involved. As they see, say, hear and touch using our signature TouchPoints on the numerals, they easily make the connection between the numeral and the quantity it represents.

### TouchPoints

TouchPoints are unique to the TouchMath Program and provide these advantages:

1. Children enjoy a multisensory math experience while keeping their attention focused on their papers
2. Older students and students with learning differences can use TouchPoints as a portable, invisible tool for as long as needed
3. Students arrive at the correct answer without guessing
4. TouchPoints help develop comprehension and memorization

### Visual Cues

The TouchMath Program uses visual cues such as arrows, boxes and dotted lines to guide students to the correct answer and promote good habits. Visual cues help to:

1. Develop left/right directionality
2. Reduce number reversals
3. Understand place value
4. Simplify and clarify all areas of computation

Tens	Ones
7	5
+	1
7	7
<hr/>	
9	2

Tens	Ones
7	8
-	5
3	3
<hr/>	
2	5

### Step-by-Step Strategy

Our sequential strategy is structured so that students fully understand one skill before we introduce the next. They feel optimistic and confident as they assimilate new information. You save the time and frustration of teaching and re-teaching.

### Age-Appropriate Materials

Our designers are acutely aware of the importance of building student self-esteem as we deliver information. For young children, this means plentiful workspace, a manageable number of problems on each page, large print, instructions that do not require reading skills, coloring pictures and games. Materials for older students of varying ability levels use fewer pictures and age-appropriate content.

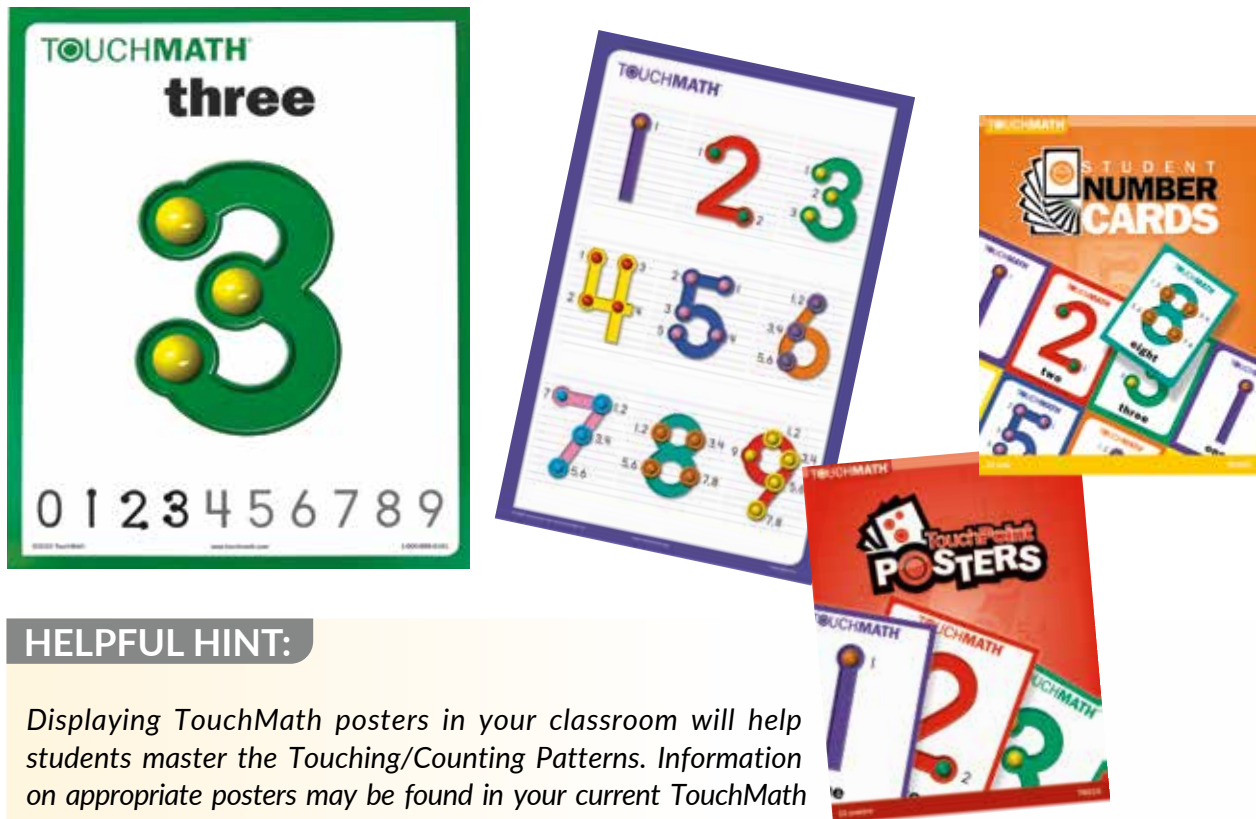
# TouchPoint Placement



Each TouchMath numeral has a corresponding number of TouchPoints placed upon the numeral. This TouchPoint placement has been carefully researched. It is important to use the same pattern consistently.

- Each numeral from 1 through 9 has TouchPoints corresponding to the digit's value.
- Numerals 1 through 5 have single TouchPoints that are touched and counted once.
- Numerals 6 through 9 have double TouchPoints that are touched and counted twice.

Teach your students to count aloud as they touch the single TouchPoints once and double TouchPoints twice. This multisensory approach engages students on auditory, visual, and tactile/kinesthetic levels. To ensure that students arrive at the right answer, it is important that they touch the TouchPoints in the correct Touching/Counting Pattern for each numeral. The Touching/Counting Patterns are shown on the next page.



## HELPFUL HINT:

Displaying TouchMath posters in your classroom will help students master the Touching/Counting Patterns. Information on appropriate posters may be found in your current TouchMath catalog or at [www.touchmath.com](http://www.touchmath.com)

## METHODOLOGY

## TouchPoint Patterning

1

The one is touched at the top while counting: "One."

7

1,2  
3,4  
5,6

The seven is touched and counted from top to bottom, counting the double TouchPoints first: "One-two, three-four, five-six," followed by the single TouchPoint: "seven." The single TouchPoint can be thought of as the nose. Teachers sometimes tell young or students needing intervention to "touch him on the nose" to help them remember the final TouchPoint.

1 2

The two is touched at the beginning and the end of the numeral while counting: "One, two."

1 2 3

The three is touched at the beginning, middle, and end of the numeral while counting: "One, two, three."

1,2 3,4  
5,6 7,8

The eight is touched and counted from left to right: "One-two, three-four, five-six, seven-eight." Tell young or students needing intervention that the eight looks like a robot. Count his head first and then his body. You may also tell students the eight is touched in the same pattern as you write the letter Z.

1 2 3 4

The four is touched and counted from top to bottom on the down strokes while counting: "One, two, three, four." For the closed shaped 4, use the same square counting pattern. To help students remember the first TouchPoint, it may be referred to as being in "outer space."

2 3 5 4

The five is touched and counted in the sequential order pictured: "One, two, three, four, five." To help in remembering the fourth TouchPoint, it may be referred to as the "belly button."

9 1,2 3,4  
5,6 7,8

The nine is touched and counted from top to bottom, counting the double TouchPoints first: "One-two, three-four, five-six, seven-eight," followed by the single TouchPoint: "nine." To the tune of "Head, Shoulders, Knees, and Toes," have the students touch these body parts with both hands while singing "head, shoulders, knees, and toes, touch the nose" (with one finger). This will physically show the nine TouchPoint counting pattern.

1,2 3,4 5,6

The six begins the use of double TouchPoints which are touched and counted twice. The six is touched and counted from top to bottom: "One-two, three-four, five-six."

0

The zero has no TouchPoints, so you never touch and count zero.

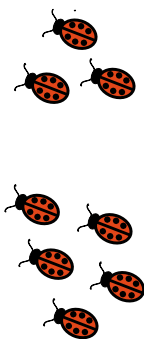
# Beginning Addition



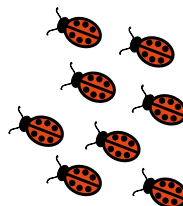
Students frequently have difficulty making the transition from the concrete (ladybugs) to the abstract ( $3 + 5$ ). To help them make the transition, place the ladybugs in the correct Touching/Counting Patterns on the numerals. Later, we replace the ladybugs with TouchPoints.

1. For Example A, count a group of 3 ladybugs: 1, 2, 3; then count a group of 5 ladybugs: 1, 2, 3, 4, 5.
2. For Example B, combine the groups to make a set of 8 ladybugs: 1, 2, 3, 4, 5, 6, 7, 8.
3. For Example C, the ladybugs are placed on the TouchPoints of the numerals.
4. Count the ladybug TouchPoints to get the answer to  $3 + 5 = 8$ .
5. You may use anything: bugs, buttons, checkers, etc. as TouchPoints on the TouchNumerals to show students TouchPoints represent the same quantity with either TouchPoints or objects.

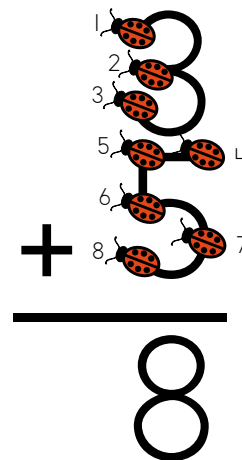
Example A






Example B






Example C





NAME \_\_\_\_\_

 +  = 

$1 + 2 = 3$

 +  = 




$1 + 2 = 3$

 +  = \_\_\_\_\_



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PK

NAME \_\_\_\_\_

 +  = 

$5 + 3 = 8$

 +  = \_\_\_\_\_

 +  = \_\_\_\_\_

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K - G2

NAME \_\_\_\_\_

$\begin{array}{r} 5 \\ + 3 \\ \hline \end{array}$  +  $\begin{array}{r} 4 \\ + 4 \\ \hline \end{array}$

$\begin{array}{r} 4 \\ + 4 \\ \hline \end{array}$  +  $\begin{array}{r} 1 \\ + 1 \\ \hline \end{array}$

$\begin{array}{r} 6 \\ + 8 \\ \hline \end{array}$  +  $\begin{array}{r} 1 \\ + 6 \\ \hline \end{array}$

$\begin{array}{r} 2 \\ + 7 \\ \hline \end{array}$  +  $\begin{array}{r} 1 \\ + 8 \\ \hline \end{array}$

There were 3 girls and 6 boys in the group. How many children were there?

\_\_\_\_\_ children

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G3 - G5

NAME \_\_\_\_\_

$\begin{array}{r} 8 \\ + 4 \\ \hline \end{array}$   $\begin{array}{r} 15 \\ - 7 \\ \hline \end{array}$   $\begin{array}{r} 17 \\ - 6 \\ \hline \end{array}$   $\begin{array}{r} 5 \\ - 8 \\ \hline \end{array}$   $\begin{array}{r} 5 \\ + 8 \\ \hline \end{array}$

$\begin{array}{r} 12 \\ - 8 \\ \hline \end{array}$   $\begin{array}{r} 16 \\ + 9 \\ \hline \end{array}$   $\begin{array}{r} 9 \\ - 7 \\ \hline \end{array}$   $\begin{array}{r} 14 \\ + 6 \\ \hline \end{array}$   $\begin{array}{r} 14 \\ - 9 \\ \hline \end{array}$

$\begin{array}{r} 15 \\ - 7 \\ \hline \end{array}$   $\begin{array}{r} 9 \\ + 8 \\ \hline \end{array}$   $\begin{array}{r} 8 \\ - 2 \\ \hline \end{array}$   $\begin{array}{r} 18 \\ - 9 \\ \hline \end{array}$   $\begin{array}{r} 16 \\ + 8 \\ \hline \end{array}$

$12 - 6 \bigcirc 15 - 9$   $6 + 1 \bigcirc 1 + 6$   
 $17 - 9 \bigcirc 12 - 8$   $5 - 2 \bigcirc 2 + 1$   
 $9 + 8 \bigcirc 13 - 9$   $4 + 8 \bigcirc 6 + 2$

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UG 6+

## ADDITION

## Beginning Addition Practice

**Step 1:** Teach the Beginning Addition Statement:

"I touch and count all the TouchPoints on the numbers."

2. Touch the TouchPoints with a pencil point and count aloud in the correct order.
  3. For Example A, begin counting with the top number in the column, 5: "1, 2, 3, 4, 5."
  4. Continue counting on the 4: "6, 7, 8, 9."
  5. Record the answer: 9.
  6. Reinforce the facts by repeating the problem and the answer aloud.\*
  7. For a longer column of numbers, Example B, repeat the first four steps and continue counting on all of the numbers. Then write the answer.
  8. Follow the same process for Examples C and D for problems in a horizontal format.
- \* This step engages auditory learning and is key in helping students memorize number facts. It is very effective for auditory learners.

Example A

$$\begin{array}{r} 5 \\ + 4 \\ \hline 9 \end{array}$$

Example B

$$\begin{array}{r} 8 \\ 2 \\ 4 \\ + 6 \\ \hline \end{array}$$

Example C

$$9 + 6 =$$

Example D

$$7 + 3 + 9 + 4 =$$



## Addition With Counting on Practice

### Step 1: Teach the Addition Counting On Statement:

"I touch the greater(est) number, say its name, and continue counting on the TouchPoints of the other number(s)."

- For Example A, touch the greater number. Say its name, "8."
- Continue counting on the TouchPoints of the 4: "9, 10, 11, 12."
- Record the answer: 12.
- Reinforce the addition facts by repeating the problem and answer aloud.
- For long columns of numbers (Example B), cross out the greatest number as you say its name, and continue counting from the top downward. Then write the answer.\*
- Solve Examples C and D for additional practice.

\* This step prevents students from counting a number twice.

Example A

$$\begin{array}{r} 8 \\ + 4 \\ \hline 12 \end{array}$$

Example B

$$\begin{array}{r} 5 \\ 3 \\ + 7 \\ \hline \end{array}$$

Example C

$$\begin{array}{r} 9 \\ 1 \\ 0 \\ 5 \\ + 7 \\ \hline \end{array}$$

Example D

$$5 + 4 + 6 + 2 =$$

## ADDITION

# Addition Without Regrouping Practice

**Step 1: Teach the Arrow Statement:**

"I start on the side with the arrow. The arrow is in the ones column on the right side."

2. For Example A, say the greater number in the ones column: "6," and continue counting on the TouchPoints of the 2: "7, 8."
3. Record the answer: 8.
4. Repeat step 2 in the tens column: "4", and continue counting on the TouchPoints of the 3: "5, 6, 7."
5. Record the answer: 7.
6. Repeat the problem and answer aloud.\*
7. For Example B, start on the side with the arrow, the ones (O) column. Then move to the tens (T) column, and then the hundreds (H) column, recording your answer for each column.
8. Solve Example C using the previous steps. Then move to the thousands (TH) column.

\* This step reinforces the ability to read and recognize large numbers.

**Example A**

Tens	Ones
	↓
4	2
+	3
3	6
78	

**Example B**

H	T	O
		↓
4	2	6
+	3	7
3	7	0

**Example C**

TH	H	T	O
			↓
2	8	6	4
+	7	1	3
7	1	3	5

# Addition With Regrouping Practice

## Step 1: Teach the Regrouping Statement:

"I must regroup if my answer is greater than 9."

2. Introduce the regrouping box. Students will write the regrouped number of tens in the box.
3. For Example A, begin in the ones column below the arrow. Say the name of the greater number "8," and continue counting on the 7: "9-10, 11-12, 13-14, 15."
4. Use the box to record the number of tens regrouped to the tens column. Record the number of tens, 1, in the box first. Then record the number of ones, 5.\*
5. Add the tens column starting at the top. You may also start with the greatest number, cross it out, and continue counting from the top: "1, 2, 3, 4, 5, 6."
6. Record the answer: 6.
7. For long columns, Example B, repeat steps 1-6.
8. Solve for Examples C and D for 3- and 4-column problems following the same steps for the hundreds (H) and thousands (TH) columns.

\* This step is important to reduce number reversals. If students write the 5 before writing the regrouped 1 they are writing the number 51 instead of 15.

Example A

Tens	Ones
	↓
2	7
+	3
3	8
<hr/>	
6	5

Example B

Tens	Ones
	↓
4	4
3	6
+	7
7	5
<hr/>	

Example C

H	T	O
		↓
4	2	6
+	3	5
3	5	4
<hr/>		

Example D

TH	H	T	O
			↓
8	3	7	0
+	4	6	9
4	6	9	5
<hr/>			

# Backward Counting Practice

## The Importance of Backward Counting:

- Builds familiarity with the entire number system
- Based on counting — the easiest of math skills
- Basis for subtraction and clarifies the relationship with addition
- Builds beginning awareness that all computation processes are different ways of working forward or backward on a number line

Students will need to count backward from 18 and every number below 18. Practice counting backward from any number and stopping at any number. For example, when counting backward from 6, stop at 5, 4, 3, 2, 1, or 0. When teaching backward counting from the numbers 6 to 18, practice with one number in each lesson until students are proficient with counting backward from that number then continue to the next number.

## Suggested Activities:

1. For practice in backward counting, one activity you can do is to say, "I'm thinking of the number 13. Count backward from 13 to 6." Begin by using a number line and progress to using no visual cues.
2. Physical activities such as walking backward, bouncing a ball, tossing a beanbag, etc., while counting backward may help reinforce the skill.
3. As children are leaving the classroom for lunch or recess, begin with a number such as 15 and have each child say a number, counting backward as he or she exits.

\* **Note:** In subtraction, you never have to count backward from any number higher than 18. Look at the examples. You can subtract any number from 9; so in the problem  $49 - 29$ , subtract 9 from 9 and get 0.  $4 - 2 = 2$ . However, when the problem is changed to  $48 - 29$ , regrouping, or borrowing, must occur. The 4 tens become 3, 8 becomes 18, and 9 is subtracted from 18, which is the largest number from which we ever count backward for subtraction.

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Name \_\_\_\_\_

$\begin{array}{r} 71 \\ - 20 \\ \hline \end{array}$	$\begin{array}{r} 60 \\ - 31 \\ \hline \end{array}$	$\begin{array}{r} 58 \\ - 30 \\ \hline \end{array}$	$\begin{array}{r} 85 \\ - 28 \\ \hline \end{array}$	$\begin{array}{r} 47 \\ - 33 \\ \hline \end{array}$
$\begin{array}{r} 59 \\ - 39 \\ \hline \end{array}$	$\begin{array}{r} 66 \\ - 26 \\ \hline \end{array}$	$\begin{array}{r} 44 \\ - 27 \\ \hline \end{array}$	$\begin{array}{r} 56 \\ - 39 \\ \hline \end{array}$	$\begin{array}{r} 73 \\ - 19 \\ \hline \end{array}$

If I subtract 44 - 24, then I ☐ need to regroup. ☐ don't need to regroup.

If I subtract 54 - 54, then I ☐ need to regroup. ☐ don't need to regroup.

If I subtract 50 - 22, then I ☐ need to regroup. ☐ don't need to regroup.

If I subtract 64 - 28, then I ☐ need to regroup. ☐ don't need to regroup.

If I subtract 76 - 19, then I ☐ need to regroup. ☐ don't need to regroup.

If I subtract 48 - 33, then I ☐ need to regroup. ☐ don't need to regroup.

If I subtract 82 - 48, then I ☐ need to regroup. ☐ don't need to regroup.

If I subtract 91 - 19, then I ☐ need to regroup. ☐ don't need to regroup.

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### Example A

Tens	Ones
4	9
- 2	9
<hr/>	
2	0

### Example B

Tens	Ones
3	8
- 2	9
<hr/>	
1	9

# Beginning Subtraction Practice



## Step 1: Teach the Beginning Subtraction Statement:

"I touch the first number, say its name, and count backward on the TouchPoints of the other number."

2. For Example A, touch the top number and say its name: "7."
3. Count backward on the TouchPoints of the 5: "6, 5, 4, 3, 2."
4. The answer is the last number spoken: "2."
5. Record the answer: 2.
6. Reinforce the subtraction facts by repeating the problem and the answer aloud.\*
7. For Example B, follow steps 1 through 6 counting backward from 14 while touching the TouchPoints on the 6.
8. Follow the same process for Examples C and D for problems in a horizontal format.

\* This step engages auditory learning and is key in helping students memorize number facts.

Example A

$$\begin{array}{r} 7 \\ - 5 \\ \hline 2 \end{array}$$

Example B

$$\begin{array}{r} 14 \\ - 6 \\ \hline \end{array}$$

Example C

$$8 - 3 =$$

Example D

$$12 - 5 =$$



SUBTRACTION

# Subtraction Without Regrouping Practice

**Step 1:** Teach Double-Digit Subtraction by Starting with the Arrow Statement:

"I start on the side with the arrow. The arrow is in the ones column on the right side."

2. For Example A, touch the top number, say its name: "7."
3. Count backward on the TouchPoints of the 4: "6, 5, 4, 3."
4. The answer is the last number spoken: "3."
5. Record the answer: 3.
6. Repeat the process in the tens column.
7. Repeat the problem and answer aloud.\*
8. Introduce a three-digit problem (Example B) by repeating the steps above, moving from the ones (O) column to the tens (T) column, then to the hundreds (H) column. Record the answer for each column.

9. Solve Example C using the previous steps. Then move to the thousands (TH) column.

\* This step reinforces the ability to read and recognize large numbers.

Example A

Tens	Ones
	↓
6	7
- 2	4
<hr/>	
4	3

Example B

H	T	O
		↓
7	9	6
- 3	2	5
<hr/>		

Example C

TH	H	T	O
			↓
5	8	2	9
- 3	6	0	7
<hr/>			

# Subtraction With Regrouping Practice

**Step 1:** Teach the Subtraction Regrouping Statement:

“If I cannot count all of the TouchPoints, I must regroup.”

2. Introduce the regrouping bar. Students will write the new number of tens on the bar after regrouping.
3. For Example A, count backward in the ones column from 3: “2, 1, 0.” Students will discover they cannot count all the TouchPoints on the 6.
4. Regroup, or borrow, one ten from the tens column. Cross out the top number in the tens column, 8, and write the regrouped number, 7, on the bar.
5. Write the regrouped ten in front of the 3 in the ones column, making it as large as the 3. Then count backward from the regrouped number, 13: “12, 11, 10, 9, 8, 7.”
6. Record the answer: 7.
7. Subtract the tens column: “7, 6, 5, 4.”
8. Record the answer: 4.
9. Solve Example B using the same steps as above.
10. Solve Examples C and D using the previous steps then move to the hundreds (H) and thousands (TH) columns

Example A

Tens	Ones
8	3
- 3	- 6
4	7

Example B

Tens	Ones
6	2
- 4	- 5

Example C

H	T	O
6	2	7
- 2	- 8	- 4

Example D

TH	H	T	O
2	8	7	4
- 1	- 9	- 2	- 5

## SKIP COUNTING

### Skip Counting Practice

#### The importance of Skip Counting:

- Based on counting—the easiest of math skills
- Basis for multiplication, basis for division, foundation for fractions, foundation for algebra
- Builds familiarity with the entire number system
- Only ten numbers in each sequence. When you count by 2s, you are counting every second number, by 3s every third number, etc.

Skip (or sequence) counting is a critical skill for multiplication, division and higher mathematical development. Students will need auditory stimulation, visual patterning cues and tactile reinforcement to help master this skill. Mastery of skip counting will lead to an understanding of fractions and build a framework for basic algebra.

#### Suggested Activities:

1. Teach one sequence at a time. Start with the 2s sequence: 2, 4, 6, ... to 20. Teach each sequence for the 2s, 3s, ... through 9s.
2. Children can be asked to skip count on their way in/out of the door to recess, music, lunch, or a restroom break.
3. Other activities such as Bean Bag Toss, Jump Rope, Hop Scotch are suggested in the skip counting materials.

#### Example A

$$5 = \boxed{5}$$

$$5 + 5 = \boxed{\phantom{00}}$$

$$5 + 5 + 5 = \boxed{\phantom{00}}$$

$$5 + 5 + 5 + 5 = \boxed{\phantom{00}}$$

$$5 + 5 + 5 + 5 + 5 = \boxed{\phantom{00}}$$

$$5 + 5 + 5 + 5 + 5 + 5 = \boxed{\phantom{00}}$$

$$5 + 5 + 5 + 5 + 5 + 5 + 5 = \boxed{\phantom{00}}$$

$$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = \boxed{\phantom{00}}$$

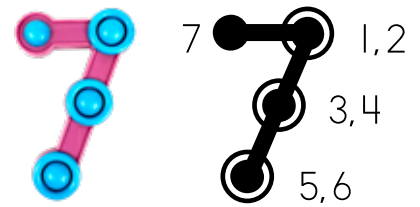
$$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = \boxed{\phantom{00}}$$

#### Example B

2	4	6	8	10	12	14	16	18	20
---	---	---	---	----	----	----	----	----	----

#### Example C

5	35	50	10	40	15	25	30	45	20
---	----	----	----	----	----	----	----	----	----



The seven is touched and counted from top to bottom, counting the double TouchPoints first: "One-two, three-four, five-six," followed by the single TouchPoint: "seven." The single TouchPoint can be thought of as the nose. Teachers sometimes tell young or students needing intervention to "touch him on the nose" to help them remember the final TouchPoint.

# Beginning Multiplication Practice



**Step 1:** Teach the Beginning Multiplication Statement:

“I skip count by one number while touching the TouchPoints on the other number.”

2. For Example A, skip count by 2s while touching the TouchPoints on the 6: “2–4, 6–8, 10–12.”
3. Record the answer: 12.
4. Reinforce facts by repeating the problem and answer aloud.\*
5. In Example B, repeat the multiplication process counting by 4s on the TouchPoints of the 5.
6. Follow the same process for Example C for problems in horizontal format.
7. Multiplication problems without the TouchPoints, Example D, allow students to respond to a multiplication problem in different ways. If the student needs to, they can add tiny Touchpoints, etc. Students may remember the fact and not use the TouchMath strategy, or they may count by the number of their choice on the other number. Often students will learn to “touch” the points visually in order to “figure out” a product.

\* This step engages the auditory senses and is key in helping students memorize number facts.

Example A

$$\begin{array}{r} 6 \\ \times 2 \\ \hline 12 \end{array}$$

Example B

$$\begin{array}{r} 5 \\ \times 4 \\ \hline \end{array}$$

Example C

$$3 \times 5 =$$

Example D

$$9 \times 6 =$$

## MULTIPLICATION

# Multiplication Without Regrouping Practice

**Step 1:** Teach Multiplication Without Regrouping by Starting with the Arrow Statement:

“I start on the side with the arrow. The arrow is in the ones column on the right side.”

- For Example A, skip count by 3s while touching the TouchPoints on the 2: “3, 6.”
- Record the answer: 6.
- Skip count by 3s while touching the TouchPoints on the 3: “3, 6, 9.”
- Record the answer: 9.
- In Example B, multiply the ones (O) column and the tens (T) column. Then move to the hundreds (H) column.
- Solve Example C using the previous steps. Then move to the thousands (TH) column.

Example A

Tens	Ones
3	2
×	3
<hr/>	
9	6

Example B

H	T	O
5	3	2
×		3
<hr/>		

Example C

TH	H	T	O
5	3	4	1
×			2
<hr/>			



# Multiplication With Regrouping Practice

## Step 1: Teach the Regrouping Statement:

“I must regroup if my answer is greater than 9.”

- For Example A, skip count by 8s while touching the TouchPoints of the 7: “8–16, 24–32, 40–48, 56.”
- Write the 5 tens in the box first. Then write the 6 ones in the ones column.\*
- Count by 8s while touching the TouchPoints on the 6: “8–16, 24–32, 40–48.”
- Add the tens in the box: 49, 50, 51, 52, 53.
- Record the answer: 53.
- In Example B, repeat the process as you multiply by 5. Then move to the hundreds (H) and thousands (TH) columns.
- In Example C, the gray shading is the visual cue for multiplying by the 6 in the tens column.

\* This step is important to reduce number reversals. If students write the 6 before writing the regrouped 5, they are writing the number 65 instead of 56.

Example A

Tens	Ones
5	
↓	
6	7
×	8
<hr/>	
5	36

Example B

TH	H	T	O
↓			
2	5	4	6
×			5
<hr/>			

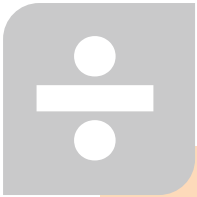
Example C

H	T	O
↓		
7	8	4
×		65
<hr/>		
+		
		0

## HELPFUL HINT

When multiplying, the student may select either the multiplier or multiplicand to skip count by. However, it is best to begin teaching multiplication with consistent usage of the multiplier.

## DIVISION



## Beginning Division Practice

**Step 1: Teach the Division Statement:**

"I skip count by the divisor and get as close to the dividend as possible without going over the dividend."

2. For Example A, skip count by 2s: "2, 4, 6, 8."
3. As each number is counted, make a tally mark in the box above the division problem.
4. Count the tally marks.
5. Record the answer: 4.
6. In Example B, repeat the process as you skip count by 6s.
7. For Examples C and D, multiply the answer (quotient) by the divisor and write that answer below the dividend, then subtract. This step prepares students for long division.

Example A

$$\begin{array}{r} \boxed{////} \\ 2 \overline{) 8} \end{array}$$

Example B

$$\begin{array}{r} \boxed{\phantom{00}} \\ 6 \overline{) 18} \end{array}$$

Example C

$$\begin{array}{r} \boxed{\phantom{00}} \\ 5 \overline{) 45} \\ \times \\ - \phantom{00} \end{array}$$

Example D

$$\begin{array}{r} \boxed{\phantom{00}} \\ 7 \overline{) 49} \\ \times \\ - \phantom{00} \end{array}$$

## HELPFUL HINT

*It is possible to teach multiplication and division simultaneously. Once the students can multiply by 6, they are able to divide by 6!*

## Division With Remainders Practice

### Step 1: Teach the Division With Remainders Statement:

"I skip count by the divisor and get as close to the dividend as possible without going over the dividend. Then I continue counting by one up to the dividend."

- For Example A, skip count by 3s.
- As each number is counted, make a tally mark in the box. Come as close to 23 as you can without going over: "3, 6, 9, 12, 15, 18, 21."
- To find the remainder, continue counting forward by 1s. Put dots outside of the box for each number counted: "22, 23."
- Count the tally marks, and record the answer (quotient): 7. Count the dots. Record the remainder: 2.
- In Example B, repeat the division process dividing by 5.
- For Examples C and D, multiply the answer (quotient) by the divisor and write that answer below the dividend, then subtract. This step prepares students for long division.

Example A

Example A shows a box with 7 tally marks (four vertical lines and three horizontal lines) and two dots to the right. Below the box is the number 7 with a subscript 'r' and the number 2. To the left of the box is the division problem  $3 \overline{)23}$ .

Example B

Example B shows a box with a horizontal line. Below the box is the number 5 with a subscript 'r' and the number 8. To the left of the box is the division problem  $5 \overline{)28}$ .

Example C

Example C shows a box with a horizontal line. Below the box is the number 6 with a subscript 'r' and the number 7. To the left of the box is the division problem  $6 \overline{)47}$ . Below the division problem is a horizontal line for multiplication and subtraction.

Example D

Example D shows a box with a horizontal line. Below the box is the number 7 with a subscript 'r' and the number 8. To the left of the box is the division problem  $7 \overline{)68}$ . Below the division problem is a horizontal line for multiplication and subtraction.

### HELPFUL HINT

TouchMath's Computation Step Posters set of six, 11" x 17" posters that reinforce addition, subtraction, multiplication, and division statements.



# Long Division Practice

1. For Example A, divide the 4 in the hundreds column by 3, and record the answer.
2. Multiply 3 by 1, and record the answer below the 4.
3. Subtract 3 from 4, and record the answer.
4. Bring down the 6 in the tens column.
5. Divide 16 by 3, and record the answer.
6. Multiply 3 by 5, and record the answer.
7. Subtract 15 from 16, and record the answer.
8. Bring down the 0 in the ones column.
9. Divide 10 by 3, and record the answer.
10. Multiply 3 by 3, and record the answer.
11. Subtract 9 from 10, and record the answer, which is the remainder.

Example A

$$\begin{array}{r}
 \times \overline{153} \text{ r } \phantom{00} \\
 3 \overline{)460} \\
 \underline{-3} \phantom{00} \\
 16 \phantom{00} \\
 \underline{-15} \phantom{00} \\
 10 \phantom{00} \\
 \underline{-9} \phantom{00} \\
 1
 \end{array}$$

Example B

$$\begin{array}{r}
 \times \overline{\phantom{000}} \text{ r } \phantom{00} \\
 8 \overline{)934} \\
 \underline{\phantom{00}} \phantom{00} \\
 \phantom{00} 3 \phantom{00} \\
 \underline{\phantom{00}} \phantom{00} \\
 \phantom{00} 4 \phantom{00} \\
 \underline{\phantom{00}} \phantom{00} \\
 \phantom{00} \phantom{00}
 \end{array}$$



## Short Division Practice

1. For Example A, divide the 4 in the hundreds column by 3, and record the answer: 1.
2. Mentally subtract 3 from 4, and write the 1 in front of the 6 in the tens column.
3. Divide 16 by 3. Write the answer: 5.
4. Mentally subtract 15 from 16. Write the 1 in front of the 0 in the ones column.
5. Divide 10 by 3. Write the answer: 3. Then write the remainder: 1.
6. For Example B, repeat the process dividing by 8.

Example A

$$\begin{array}{r} \times \phantom{00} | 5 | 3 \text{ r } | \\ 3 \overline{) 4 | 6 | 0} \end{array}$$

Example B

$$\begin{array}{r} \times \phantom{00} | | | \\ 8 \overline{) 9 | 3 | 4} \end{array}$$

## Money Kit



## Money Practice



### Money Kit

Introduce coins, bills, and build strong financial literacy life skills the TouchMath way. This extra practice kit extends the TouchMath curriculum with 285 student activities, instructional support, and a plethora of teaching aids. Print and Digital Solution options available.

### Money Program Outcomes:

- Identify coins and their values
- Sort, classify, and order coins by size, value, and appearance
- Sort, classify, and order bills by value
- Determine the total amount of coins and bills
- Determine the costs of items and make change
- Develop a variety of problem solving strategies
- Collect, organize, and display data using tables, charts, and graphs
- Using words, students will write money values
- Add, subtract, multiply, and divide using dollar signs and decimal points
- Estimate and round money values
- Do real-life activities with bills, coins, bank accounts, menus, and shopping with coupons

### Suggested Activities:

1. To introduce each new coin, use real coins whenever possible. Students make an instant connection when they see real coins alongside those with TouchPoint and TouchStar cueing.
2. Make sure students can skip count by 5s before they begin to learn the values of the nickel, dime, and quarter.
3. Use the Money Wheel pie charts to show how many pennies, nickels, dimes, quarters, and half-dollars it takes to make one dollar. Have students place plastic coins on top of the coins pictured on the pie charts for a hands-on activity to aid the learning process.
4. Set up a classroom grocery store with various items. Place a price tag on each item using amounts that are age/ability appropriate. Have students shop and pay for the items using play money. Coupons may be included for more advanced students.
5. Set up a classroom restaurant with age-appropriate menus. Have students choose items to order from the menu and calculate the totals. Then have the "customers" pay the "cashier" using play money.

## Penny and Nickel Counting Patterns

### Step 1: Teach the TouchPoint Statement:

"I count by 1 on the TouchPoint."

1. When you see a TouchPoint on a penny, touch and count by 1, like counting TouchPoints on numbers.



### Step 2: Teach the TouchStar Statement:

"I count by 5 on the TouchStar."

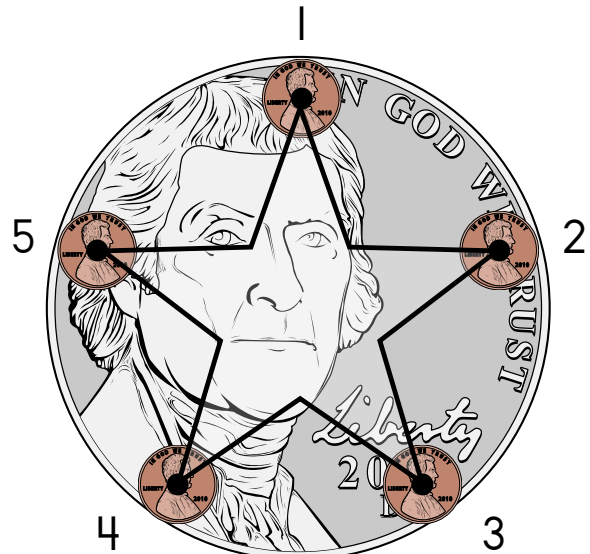
2. The TouchStar has 5 points. When you see a TouchStar, touch and count by 5.
3. 1 nickel equals 5 pennies. 1 nickel equals 5 cents. A TouchStar equals 5 pennies or 5¢.



5



5

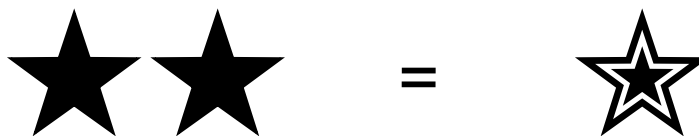


# Dime Counting Pattern

**Step 1:** Teach the Double TouchStar Statement:

“I count by 5 on the TouchStar. Double TouchStars are touched and counted twice.”

2. When you see a double TouchStar, touch and count by 5s; 5, 10.
3. 1 dime equals 2 nickels. 1 dime equals 10 cents. 2 TouchStars equals 10 pennies or 10¢.



5, 10



5, 10



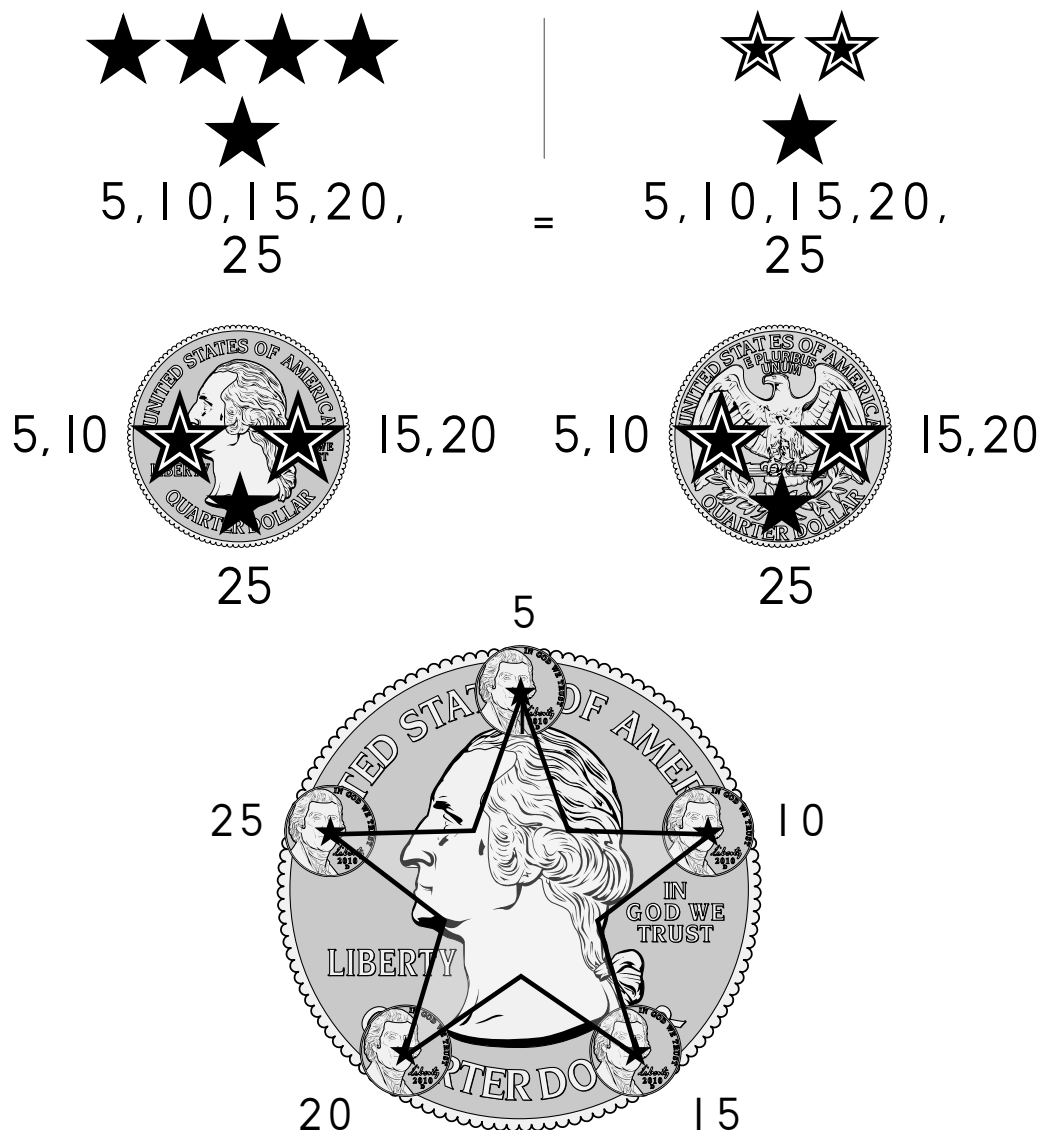


## Quarter Counting Pattern

### Step 1: Teach the Double TouchStar Statement:

"I count by 5 on the TouchStar. Double TouchStars are touched and counted twice."

- The quarter has two double TouchStars and a single TouchStar. Begin counting on the double TouchStars; 5-10, 15-20, 25.
- 1 quarter equals 5 nickels. 1 quarter equals 25 cents. 5 TouchStars equals 25 pennies or 25¢.



Money Kit

# Money Practice

## Valuing Coins


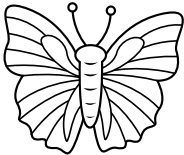



Counting on the TouchStars and TouchPoints makes determining the value of coins fast, accurate, and easy.



## Making Change

Counting forward on the TouchPoints and TouchStars simplifies the process of teaching students to make change.

1. You have one quarter. Say the cost of the sticker and count forward to 25. As you count, ring the coins needed. Then cross out the coins not needed. Now add the value of the correct coins and write the amount of **change** returned.

Money Paid	Sticker Bought	Change
	 7¢	 8¢ 9¢ 10¢  15¢  25¢ <div style="text-align: right;">18¢ change</div>

## Computation

TouchMath Money also incorporates the program's signature TouchPoints when beginning money computation.

$\begin{array}{r} \downarrow \\ \$6.52 \\ + 1.34 \\ \hline \$ \quad . \end{array}$	$\begin{array}{r} \downarrow \\ \$2.45 \\ + 4.50 \\ \hline \$ \quad . \end{array}$	$\begin{array}{r} \downarrow \\ \$7.55 \\ - 4.11 \\ \hline \$ \quad . \end{array}$	$\begin{array}{r} \downarrow \\ \$8.56 \\ - 3.34 \\ \hline \$ \quad . \end{array}$
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# Identifying and Renaming Fractions

 $\frac{1}{2}$ 

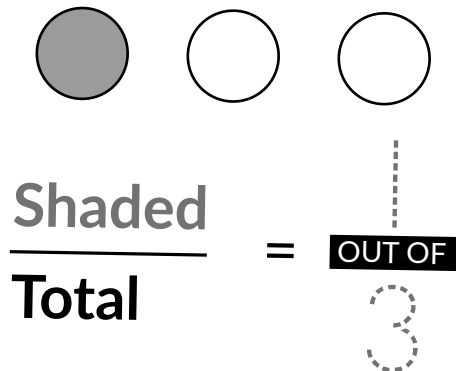
The process of teaching fractions is done using TouchMath's scaffolded approach by introducing each new skill with an explanation and step-by-step directions and visual examples. Your students will master fractions in a fraction of the time.

## Identifying fractions

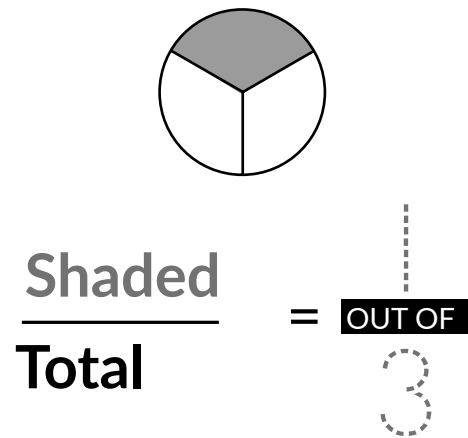
TouchMath combines pictorial examples with explanations when introducing fraction concepts as parts of a set or a whole.

1. The number above the line segment shows the number shaded. The number below the line segment shows the total number. The fraction shows  $\frac{1}{3}$  or 1 out of 3.

### Example A



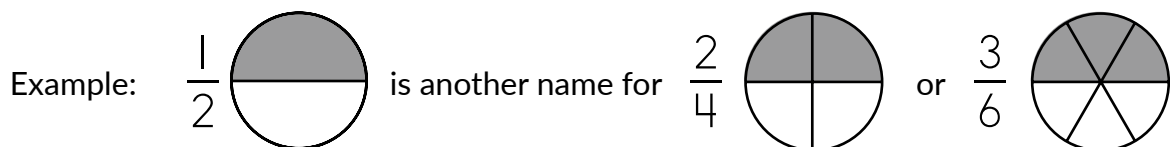
### Example B



## Renaming fractions

When renaming a fraction, its numbers change, but not its value. The fractions are equal, but have different names.

1. This is easily understood by the examples TouchMath uses to demonstrate fraction concepts.



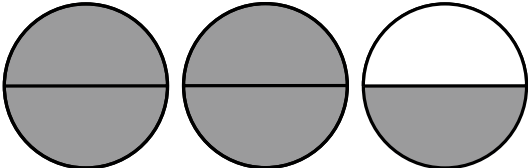
# Converting Improper Fractions

TouchMath skills previously learned are incorporated into more difficult fraction problems. When converting improper fractions to a whole or mixed number, the students will use the previously learned division process.

An improper fraction shows more parts than one whole. To convert an improper fraction to a whole or mixed number, divide the numerator by the denominator.

1. Convert each fraction below to a whole or mixed number.

**Example A**

$\frac{5}{2} =$    $=$  2 wholes and  $\frac{1}{2}$

**Example B**

$\frac{9}{7} =$  1  $\frac{2}{7}$

**Example C**

$\frac{8}{3} =$

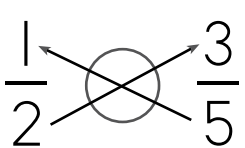
# Cross Multiplying Fractions

To discover which fraction is greater without using a chart, you may cross multiply the fractions. This is a shortcut for converting the fractions to a common denominator without writing the denominator.

1. Write the sign to show which fraction is greater than (>), less than (<), or equal to (=). Place TouchPoints on the numerators if students need them to multiply

► Cross multiply the numerators by the denominators and write the product in the boxes.

$2 \times 3 = 6$  and  $5 \times 1 = 5$ .

$\frac{1}{2}$    $\frac{3}{5}$

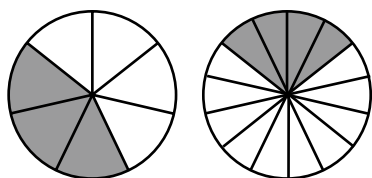
► The common denominator is 10 ( $2 \times 5$ ) so you have  $\frac{5}{10}$  and  $\frac{6}{10}$ .

► 5 is less than 6 ( $5/10 < 6/10$ ) so  $1/2$  is less than  $3/5$ .

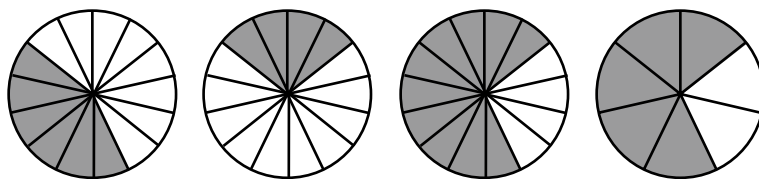
## Adding Fractions

To add fractions, they must have the same denominators. Find the common denominator and add the fractions. To add fractions you only add the numerators, the denominator stays the same. Rename the answer to lowest terms if necessary. Place TouchPoints on the numerator if your students need them to add.

1. Choose the fraction with the greatest denominator.
2. Write that fraction below the original.
3. Rename the other fraction to the common denominator and write that fraction below the original.
4. Add the fractions and write the answer.
5. Rename the answer to lowest terms if necessary.



$$\frac{3}{7} + \frac{4}{14} =$$



$$\frac{6}{14} + \frac{4}{14} = \frac{10}{14} = \frac{5}{7}$$



# Multiplying Fractions

To multiply fractions, you multiply the numerators, then the denominators. Rename the answer to lowest terms if necessary.

1. Use the TouchMath steps for multiplication if your students need them.

$$\frac{1}{3} \times \frac{9}{16} = \frac{9}{48} = \frac{3}{16}$$

► Multiply the numerators and the denominators

► Divide the answer by 3 to rename the answer to lowest terms

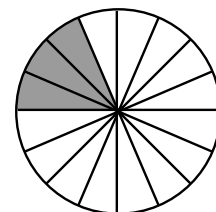
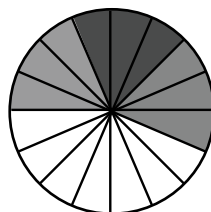
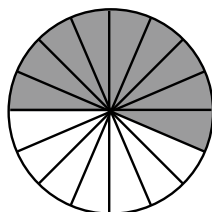
## Multiplying fractions explained:

TouchMath gives students an explanation for what they are doing.

1. For the problem  $\frac{1}{3} \times \frac{9}{16}$ , what you really want to know is: How much is  $\frac{1}{3}$  of  $\frac{9}{16}$ ?

$$\frac{1}{3} \times \frac{9}{16} =$$

► Divide  $\frac{9}{16}$  into thirds



$$\frac{9}{16}$$

$$\frac{3}{3} \text{ of } \frac{9}{16}$$

$$\frac{1}{3} \text{ of } \frac{9}{16} = \frac{3}{16}$$

# Pre-Algebra: Skip Count to Identify “x”



- To solve the number sentence you may skip count to solve the first part of the equation.

## Example A

$$3 \times 8 \bigcirc 22$$

Think " $3 \times 8 = 24$ "

$$24 > 22$$

24 is **greater than** 22

## Example B

$$4 \times 7 \bigcirc 35$$

The letter “x” can stand for any number. When a numeral and an “x” are written next to each other, it means multiply.

- In the equations below, the letter “x” is equal to 5. To find the answer you multiply 4 x 5. Skip count by 4s on the 5: 4, 8, 12, 16, 20. The answer is 20.

If  $x = 5$   
 Then  $4x = \underline{20}$   
 (Think: **4e%**)

If  $x = 5$   
 Then  $5x = \underline{\quad}$   
 (Think: **5e%**)

In the equations below, the letter “x” is unknown.

To find the answer you must divide 8 by 2. Skip count by 2s up to 8 making a tally mark as you say each number: 2, 4, 6, 8. Then count the tally marks and write the answer: 4. Check your answer by multiplying  $2 \times 4$ . Skip count by 2s on the 4: 2, 4, 6, 8.

## Example A

If  $2x = 8$   
 Then  $x = \underline{4}$  ////  
 (Think: **8r2**)

Check:

$$\underline{2} \times \underline{4} = \underline{8}$$

## Example B

If  $2x = 4$   
 Then  $x = \underline{\quad}$     
 (Think: **4r2**)

Check:

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

METHODOLOGY

The Frayer Model

Definition in your words	Facts/characteristics
Examples	Nonexamples

Name: \_\_\_\_\_



Problem or Word Problem:		
<b>Build It</b> Use a manipulative that you feel most comfortable with to show this problem and solve it.	<b>Draw It</b> Draw a picture of this problem and solve it.	<b>Write It</b> Write a number sentence and solve it.

# TOUCHMATH<sup>®</sup>

## CERTIFICATE OF COMPLETION

ON THIS \_\_\_\_ DAY OF \_\_\_\_ IN THE YEAR \_\_\_\_

HAS COMPLETED PROFESSIONAL DEVELOPMENT LESSONS, DISCUSSIONS, AND ACTIVITY PAGES FOR THE  
**TOUCHMATH PROGRAM FOR MATHEMATICS INSTRUCTION**

TO INCLUDE \_\_\_\_ HOURS.

*S. Elliott*

Sandra Elliott, Ph. D  
TouchMath Chief Academic Officer

\_\_\_\_\_  
INSTRUCTOR



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# PROFESSIONAL DEVELOPMENT

At TouchMath, we believe everyone can learn math. Our unique and individualized approach to designing professional development and training plans enables us to meet the unique needs of each partnership. With a diverse team of professionals who draw from their own classroom experiences, the most current research, and evidence-based instructional practices, we are here to help drive measurable math change for your program and students.

Easy to  
Teach and  
Easy to  
Learn

## Jumpstart

An overview of the TouchMath methodology and how to use the TouchMath Solution in the classroom. Available virtually and in person, Jumpstart is tailored to meet the unique needs of each partnership.

## Classroom Coaching

Classroom coaches provide expert advice on program implementation, the learning environment, instructional strategies, data collection, and so much more! Each coaching session is individualized and customized to meet the unique needs of each classroom.

## Custom Training

Custom professional learning opportunities empower teachers to refine their instructional techniques and integrate innovative methods, ultimately enhancing student engagement and driving math success.

## Train-the-Trainer

Promote long-term program success and sustainability by building capacity within your district. This in-depth partnership experience is designed to immerse educators and leaders in the world of TouchMath methodology and product competency.

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Go beyond product training and foundational professional learning to focus on key topics related to evidence-based practices, special education, mathematics, and pedagogy with our expert-delivered workshops.

Learn more at [touchmath.com/professional-development](https://touchmath.com/professional-development)

